Optimizing Nonprofit Warehouse Operations under Display-Dependent Demand

1. Introduction and Problem Definition

Food banks play a critical role in alleviating hunger and reducing food waste in the United States, where millions rely on food assistance. When distributing food, many food banks use a "shopping" model that allows individuals or agencies to select their own food from a central warehouse rather than receiving pre-packed allocations. This autonomy creates an empowering experience, ensures food is distributed based on demand and better accommodates specific nutritional requirements or cultural preferences. Nonprofit managers in this setting aim to balance two objectives: ensuring equitable access to diverse resources over time (*fairness objective*) while accepting and distributing as much donations as possible within limited warehouse capacity (*efficiency objective*). Achieving this balance is difficult due to unique operational constraints.

An important, challenging feature is uncertain and *display-dependent* demand—people take more of a food type when larger quantities are displayed in the warehouse. While display-dependent demand has been studied in profit-driven settings, it has generally been secondary to pricing or order quantities as control levers. In food banks, however, managers lack control over order quantities (donations) or pricing, making display decisions the primary tool for managing inventory. Additionally, warehouse capacity is severely limited. At our partner food bank, 25% of donations must be rejected or redirected due to warehouse congestion. Space constraints are further complicated by the need to manage heterogeneous food categories—such as snacks, drinks, and staple foods—with varying demand and value. While some studies examined fair and efficient allocation in nonprofit settings, they focus on stylized models with pre-packed allocations and ignore capacity constraints and resource heterogeneity.

Traditional warehouse operations have relied on manual heuristics, leading to suboptimal decisions that can force food banks to turn away donations and damage donor relationships. During shortages, early-arriving agencies may deplete available resources, leaving later arrivals underserved—undermining the nonprofit's mission of equitable access. Our work address this novel yet important inventory management problem, fill the aforementioned gaps in literature, and offer practice-driven insights and solutions. Using data from a newly automated inventory system at a regional U.S. food bank, we develop a data-driven model that captures real-world constraints: uncertain and display-dependent demand, limited capacity, and heterogeneous resources. By characterizing optimal policies, we identify key drivers of the fairness-efficiency trade-off and propose actionable interventions. Our findings inform the ongoing development of a decision-support tool for our partner food bank to improve its warehouse operations and amplify its impact.

2. Data-driven Inventory Model

We develop an inventory model that balances fairness and efficiency in nonprofit resource allocation. The model incorporates three key challenges in nonprofit operations: uncertain and display-dependent demand, limited warehouse capacity, and multiple heterogeneous resource categories.

Modeling Framework. A nonprofit planner distributes N types of divisible resources across T periods via the shopping model, subject to a shared warehouse capacity C. At the start of each period t, the planner makes the following decisions for each resource type n: (i) how much of the donated supply $S_{t,n}$ to accept ($\boldsymbol{r_{t,n}}$) without exceeding warehouse capacity, and (ii) how much inventory $I_{t,n}$ to display ($\boldsymbol{q_{t,n}}$). Incoming donations $S \in \mathbb{R}^{T\times N}$ are known from the beginning.

Inventory Dynamics and Demand Prediction. Let $I_{t,n}$ and $I'_{t,n}$ denote the starting and ending inventory for resource type n on period t respectively. Inventory evolves as follows:

$$I_{t,n} = I'_{t-1,n} + \boldsymbol{r_{t,n}}, \quad I'_{t,n} = I_{t,n} - D_{t,n}(\boldsymbol{q_{t,n}}),$$

where the inventory outflow $D_{t,n}(\boldsymbol{q_{t,n}})$ represents stochastic demand under the display decision $\boldsymbol{q_{t,n}}$. Using inventory data from our partner food bank, we observe a strong positive correlation between available inventory and outflows, which we model as:

$$D_{t,n}(\boldsymbol{q_{t,n}}) = \beta_{t,n} \cdot \boldsymbol{q_{t,n}}, \quad \beta_{t,n} \sim F_n,$$

where $\beta_{t,n}$, the *outflow rate*, is random. For each food category in the data, we use linear regression to estimate expectation of β. An uncertainty set, $\beta \in \mathcal{U}^{N\times T}$, is constructed based on residual analysis and can be easily calibrated using inventory data in practice.

Dynamic Inventory Optimization with Dual Objective. The nonprofit planner seeks to optimize the dual objectives *display fairness* and *throughput efficiency*. *Display fairness* (v^F) is defined as the *minimum display value* across the T time periods, and throughput efficiency (v^T) is defined as *the total value of received donations* summed across the T time periods, i.e.,

$$v^F = \min_{t\in[T]} \left\{ \sum_{n\in[N]} (P_n \cdot \boldsymbol{q_{t,n}}) \right\}, \quad v^T = \sum_{t\in[T]} \sum_{n\in[N]} (P_n \cdot \boldsymbol{r_{t,n}}),$$

where P_n denote the value[1] of resource type n per unit volume. To optimize both objectives, we first maximize the *worst-case* display fairness over the uncertainty set $\beta \in \mathcal{U}^{N\times T}$ by solving the dynamic optimization problem. The *worst-case* throughput efficiency v^T is then maximized over $\mathcal{U}$ while maintaining optimal display fairness. This yields the optimal acceptance and display policy. Focusing on the worst-case over a data-driven uncertainty set ensures that the optimal policies remain robust under demand uncertainties and enables closed-form, interpretable solutions in the presence of multiple resource categories to derive analytical insights.

[1] In the context of food banks, the value of a certain food type (e.g. snacks, staple food, drinks) can be defined using its nutritional value, its scarcity, its monetary value, or a combination of all.

3. Key Results and Managerial Implications

We first derive closed-form solutions for the optimal acceptance and display policies in a two-period model with one and two resource types. Building on analytical insights, we develop fast computational algorithms to solve the general case with T periods and N resource types. Using real warehouse data from our partner food bank, we evaluate the algorithm's performance and quantify its practical impact. Our work has led to an ongoing effort to implement a decision-support tool aimed at improving warehouse operations for the food bank.

The analytical solutions for the two-period model reveal the nuanced trade-off between fairness and efficiency in nonprofit warehouse operations, offering valuable managerial insights. We define the *price of fairness* (POF) as the gap between maximum achievable throughput efficiency (T^*) and efficiency under maximal display fairness (T^F), i.e., $POF = T^* - T^F$. We fully characterize conditions under which this trade-off arises ($POF > 0$) and identify its key drivers. For a single resource type, $POF > 0$ when there is high uncertainty in the outflow rate β. To ensure fairness, a nonprofit manager may limit display quantities across periods; if outflow is slower than expected, however, this can lead to warehouse congestion and reduce throughput efficiency. With multiple resource types, the trade-off is further influenced by heterogeneity in outflow rates and resource values. When resources have comparable values, efficiency-maximizing policies prioritize fast-moving items to prevent congestion, while fairness-maximizing strategies favor slower-moving items to maintain stable inventory. If one resource is significantly more valuable, fairness-maximizing policies focus on rationing it over time, even at the cost of reduced warehouse space for lower-value donations, which impacts throughput. We further establish conditions under which *flexible flow interventions* — which allow controlled acceleration of outflow through nudging or delivery — can mitigate warehouse congestion and improve efficiency without sacrificing fairness. We show that when multiple resource types are involved, flexible flow reduces POF by addressing the efficiency-fairness trade-off due to heterogeneity in outflow rates and resource values.

Using inventory data from our partner food bank, we backtest our algorithm on data from July to December 2023. Simulations estimate over 10% improvement in both display fairness and throughput efficiency under the optimal policy with flexible flow intervention, compared to the status quo. These insights have driven the ongoing development of a decision-support tool for the food bank. The tool aims to integrate (i) a predictive component that forecasts shortages and congestion risks and (ii) an optimization component that recommends when to withhold inventory for fairness and when to accelerate outflow for efficiency for individual food categories. It also incorporates practical constraints such as separate warehouse space, portfolio constraints on minimum display quantities, and seasonality in demand. By integrating inventory modeling with real-world data and practice, our work paves the way for a scalable framework that helps nonprofits optimize their warehouse to reduce waste and improve equitable access to essential resources in underserved communities.